

On stable exponential solutions in Einstein-Gauss-Bonnet cosmology with zero variation of G

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A D -dimensional gravitational model with a Gauss-Bonnet term and the cosmological term Λ is considered. Assuming diagonal cosmological metrics, we find, for certain $\Lambda \neq 0$ new examples of solutions with an exponential time dependence of two scale factors, governed by two Hubble-like parameters $H > 0$ and $h < 0$, corresponding to submanifolds of dimensions m and l , respectively, with $(m, l) = (4, 2), (5, 2), (5, 3), (6, 7), (7, 5), (7, 6)$ and $D = 1 + m + l$. Any of these solutions describes an exponential expansion of “our” 3-dimensional factor-space with Hubble parameter H and zero variation of the effective gravitational constant G . We also prove the stability of these solutions in the class of cosmological solutions with diagonal metrics.

1 Introduction

In this paper we consider D -dimensional gravitational model with Gauss-Bonnet term and cosmological term. We note that at present the so-called Einstein-Gauss-Bonnet (EGB) gravitational model and its modifications, see [1]- [14] and refs. therein, are intensively studied in cosmology, e.g. for possible explanation of accelerating expansion of the Universe following from supernovae (type Ia) observational data [15, 16, 17].

Here we deal with the cosmological solutions with diagonal metrics governed by $n > 3$ scale factors depending upon one variable, which is the synchronous time variable. We restrict ourselves by the solutions with exponential dependence of scale factors and present six new examples of such solutions: five - in dimensions $D = 7, 8, 9, 13$ and two - for $D = 14$. Any of these solutions describes an exponential expansion of 3-dimensional factor-space with Hubble parameters $H > 0$ [18] and has a constant volume factor of internal space, which imply zero variation of the effective gravitational constant G either in Jordan or Einstein frame [19, 20], see

also [21, 22] and refs. therein. These solutions obey the most severe restrictions on variation of G [23].

We study the stability of these solutions in a class of cosmological solutions with diagonal metrics by using results of refs. [13, 14] and show that all solutions, presented here, are stable. It should be noted that two exponential solutions with two factor spaces (one of which is expanding and other one - contracting) and constant G were found for $D = 22, 28$ and $\Lambda = 0$ in ref. [11]. In ref. [13] it was proved that these solutions are stable.

2 The cosmological model

The action reads

$$S = \int_M d^D z \sqrt{|g|} \{ \alpha_1 (R[g] - 2\Lambda) + \alpha_2 \mathcal{L}_2[g] \}, \quad (1)$$

where $g = g_{MN} dz^M \otimes dz^N$ is the metric defined on the manifold M , $\dim M = D$, $|g| = |\det(g_{MN})|$, Λ is cosmological term,

$$\mathcal{L}_2 = R_{MNPQ} R^{MNPQ} - 4R_{MN} R^{MN} + R^2$$

is standard Gauss-Bonnet term and α_1, α_2 are non-zero constants.

Here we consider the manifold

$$M = \mathbb{R} \times M_1 \times \dots \times M_n \quad (2)$$

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with the metric

$$g = -dt \otimes dt + \sum_{i=1}^n B_i e^{2v^i t} dy^i \otimes dy^i, \quad (3)$$

where $B_i > 0$ are arbitrary constants, $i = 1, \dots, n$, and M_1, \dots, M_n are one-dimensional manifolds (either \mathbb{R} or S^1) and $n > 3$.

Equations of motion for the action (1) give us the set of polynomial equations [13]

$$G_{ij}v^i v^j + 2\Lambda - \alpha G_{ijkl}v^i v^j v^k v^l = 0, \quad (4)$$

$$\left[2G_{ij}v^j - \frac{4}{3}\alpha G_{ijkl}v^j v^k v^l \right] \sum_{i=1}^n v^i - \frac{2}{3}G_{ij}v^i v^j + \frac{8}{3}\Lambda = 0, \quad (5)$$

$i = 1, \dots, n$, where $\alpha = \alpha_2/\alpha_1$. Here $G_{ij} = \delta_{ij} - 1$ and $G_{ijkl} = G_{ij}G_{ik}G_{il}G_{jk}G_{jl}G_{kl}$ are, respectively, the components of two metrics on \mathbb{R}^n [9, 10]. The first one is 2-metric and the second one is a Finslerian 4-metric. For $n > 3$ we get a set of forth-order polynomial equations.

For $\Lambda = 0$ and $n > 3$ the set of equations (4) and (5) has an isotropic solution $v^1 = \dots = v^n = H$ only if $\alpha < 0$ [9, 10]. This solution was generalized in [8] to the case $\Lambda \neq 0$.

It was shown in [9, 10] that there are no more than three different numbers among v^1, \dots, v^n when $\Lambda = 0$. This is valid also for $\Lambda \neq 0$ if $\sum_{i=1}^n v^i \neq 0$ [14].

3 Solutions with constant G

In this section we present several solutions to the set of equations (4), (5) of the following form

$$v = (H, \dots, H, h, \dots, h). \quad (6)$$

where H is the ‘‘Hubble-like’’ parameter corresponding to m -dimensional subspace with $m > 3$ and h is the ‘‘Hubble-like’’ parameter corresponding to l -dimensional subspace, $l > 1$. We put $H > 0$ for a description of an accelerated expansion of 3-dimensional subspace (which may describe our Universe) and also put

$$h = -(m-3)H/l < 0 \quad (7)$$

for a description of a zero variation of the effective gravitational constant G .

We remind that the effective gravitational constant $G = G_{eff}$ in the Brans-Dicke-Jordan (or simply Jordan) frame [19] (see also [20]) is proportional to the inverse volume scale factor of the internal space, see [21, 22] and refs. therein.

According to ansatz (6), the m -dimensional subspace is expanding with the Hubble parameter $H > 0$, while l -dimensional subspace is contracting with the ‘‘Hubble-like’’ parameter $h < 0$.

For $\Lambda = 0$, $m = 11$, $l = 16$ and $\alpha = 1$ the solution with $H = \frac{1}{\sqrt{15}}$, $h = -\frac{1}{2\sqrt{15}}$, describing zero variation of G , was found in [11]. Another solution of such type with $\Lambda = 0$, $H = \frac{1}{6}$, $h = -\frac{1}{3}$ and constant G appears for $m = 15$, $l = 6$ and $\alpha = 1$ [11]. It was proved in [13] that these two solutions are stable.

Here we present three solutions with constant G for $\alpha < 0$:

$$H = \frac{1}{\sqrt{6|\alpha|}}, \quad h = -\frac{1}{2\sqrt{6|\alpha|}} \quad (8)$$

for $\Lambda = 7/(8|\alpha|)$, $(m, l) = (4, 2)$;

$$H = \frac{1}{\sqrt{8|\alpha|}}, \quad h = -\frac{1}{\sqrt{8|\alpha|}} \quad (9)$$

for $\Lambda = 17/(16|\alpha|)$, $(m, l) = (5, 2)$ and

$$H = \frac{3}{2\sqrt{10|\alpha|}}, \quad h = -\frac{1}{\sqrt{10|\alpha|}} \quad (10)$$

for $\Lambda = 177/(80|\alpha|)$, $(m, l) = (5, 3)$.

We also present three solutions with constant G for $\alpha > 0$:

$$H = \frac{7}{2\sqrt{5\alpha}}, \quad h = -\frac{3}{2\sqrt{5\alpha}} \quad (11)$$

for $\Lambda = -177.45\alpha^{-1}$, $(m, l) = (6, 7)$;

$$H = \frac{5}{6\sqrt{\alpha}}, \quad h = -\frac{2}{3\sqrt{\alpha}}, \quad (12)$$

for $\Lambda = -155/(6\alpha)$, $(m, l) = (7, 5)$, and

$$H = \frac{3}{2\sqrt{5\alpha}}, \quad h = -\frac{1}{\sqrt{5\alpha}} \quad (13)$$

for $\Lambda = -8.7\alpha^{-1}$, $(m, l) = (7, 6)$.

All six solutions may be verified by MAPLE. The derivation of a more general class of solutions will be given in a separate paper.

4 Stability analysis

In [13, 14] we restricted ourselves by exponential solutions (3) with non-static volume factor, which is proportional to $\exp(\sum_{i=1}^n v^i t)$, i.e. we put

$$K = K(v) = \sum_{i=1}^n v^i \neq 0. \quad (14)$$

We put the following restriction on the matrix $L = (L_{ij}(v)) = (2G_{ij} - 4\alpha G_{ijk}s v^k v^s)$ [13, 14]

$$(R) \quad \det(L_{ij}(v)) \neq 0. \quad (15)$$

For general cosmological setup with the metric $g = -dt \otimes dt + \sum_{i=1}^n e^{2\beta^i(t)} dy^i \otimes dy^i$, we obtained in [13, 14] the (mixed) set of algebraic and differential equations

$$f_0(h) = 0, \quad (16)$$

$$f_i(\dot{h}, h) = 0, \quad (17)$$

$i = 1, \dots, n$, where $h = h(t) = (h^i(t)) = (\dot{\beta}_i(t))$ is the set of so-called ‘‘Hubble-like’’ parameters; $f_0(h)$ and $f_i(\dot{h}, h)$ are polynomials of the fourth order in h^i ; $f_i(\dot{h}, h)$ are polynomials of the first order in \dot{h}^i , see [13, 14]. The fixed point solutions $h^i(t) = v^i$ ($i = 1, \dots, n$) to eqs. (16), (17) correspond to exponential solutions (3), which obey eqs. (4), (5).

It was proved in [14] that a fixed point solution $(h^i(t)) = (v^i)$ ($i = 1, \dots, n$; $n > 3$) to eqs. (16), (17) obeying restrictions (14), (15) is stable under perturbations $h^i(t) = v^i + \delta h^i(t)$, $i = 1, \dots, n$, (as $t \rightarrow +\infty$) if

$$K(v) = \sum_{k=1}^n v^k > 0 \quad (18)$$

and it is unstable (as $t \rightarrow +\infty$) if $K(v) = \sum_{k=1}^n v^k < 0$.

It was shown in [14] that for the vector v from (6), obeying

$$mH + lh \neq 0, \quad H \neq h, \quad (19)$$

the matrix L has a block-diagonal form

$$(L_{ij}) = \text{diag}(L_{\mu\nu}, L_{\alpha\beta}), \quad (20)$$

where

$$L_{\mu\nu} = G_{\mu\nu}(2 + 4\alpha S_{HH}), \quad (21)$$

$$L_{\alpha\beta} = G_{\alpha\beta}(2 + 4\alpha S_{hh}) \quad (22)$$

and

$$S_{HH} = (m-2)(m-3)H^2 + 2(m-2)lHh + l(l-1)h^2, \quad (23)$$

$$S_{hh} = m(m-1)H^2 + 2m(l-2)Hh + (l-2)(l-3)h^2. \quad (24)$$

The matrix (20) is invertible if and only if $m > 1$, $l > 1$ and

$$S_{HH} \neq -\frac{1}{2\alpha}, \quad S_{hh} \neq -\frac{1}{2\alpha}. \quad (25)$$

We remind that the matrices $(G_{\mu\nu}) = (\delta_{\mu\nu} - 1)$ and $(G_{\alpha\beta}) = (\delta_{\alpha\beta} - 1)$ are invertible only if $m > 1$ and $l > 1$.

The first condition (18) is obeyed for the solutions under consideration since due to (7) we get $K(v) = 3H > 0$ [14].

Now, let us verify the second condition (25). The calculations give us

$$(-2\alpha)S_{HH} = -0.5, -1, -1.5, 21, 10, 6, \quad (26)$$

$$(-2\alpha)S_{hh} = 4, 5, 6, -39, -17, -9 \quad (27)$$

for the solutions with $(m, l) = (4, 2), (5, 2), (5, 3), (6, 7), (7, 5), (7, 6)$, respectively. Thus, conditions (25) are satisfied for all our solutions. Hence all six solutions are stable in a class of cosmological solutions with diagonal metrics.

5 Conclusions

We have considered the D -dimensional Einstein-Gauss-Bonnet (EGB) model with the Λ -term. By using the ansatz with diagonal cosmological metrics, we have found, for certain $\Lambda \neq 0$ and $\alpha = \alpha_2/\alpha_1$, six new solutions with exponential dependence of two scale factors with respect to synchronous time variable t in dimensions $D = 1 + m + n$, where $(m, l) = (4, 2), (5, 2), (5, 3), (6, 7), (7, 5), (7, 6)$. Here $m > 3$ is the dimension of the expanding subspace and $l > 1$ is the dimension of contracting subspace. The first three solutions correspond to $\alpha < 0$, while other three solutions correspond to $\alpha > 0$.

Any of these solutions describes an exponential expansion of ‘‘our’’ 3-dimensional factor-space with the Hubble parameter $H > 0$ and anisotropic behaviour of $(m-3+l)$ -dimensional internal space: expanding in $(m-3)$ dimensions (with Hubble-like parameter H) and contracting in l dimensions

(with Hubble-like parameter $h < 0$). Each solution has a constant volume factor of internal space and hence it describes zero variation of the effective gravitational constant G . By using results of ref. [14] we have proved that all these solutions are stable as $t \rightarrow +\infty$.

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References

- [1] H. Ishihara, Phys. Lett. **B 179**, 217 (1986).
- [2] N. Deruelle, Nucl. Phys. **B 327**, 253 (1989).
- [3] E. Elizalde, A. N. Makarenko, V. V. Obukhov, K. E. Osetrin and A. E. Filippov, Phys. Lett. **B 644**, 1 (2007).
- [4] A. Toporensky and P. Tretyakov, Grav. Cosmol. **13**, 207 (2007).
- [5] I. V. Kirnos, A. N. Makarenko, S. A. Pavluchenko and A. V. Toporensky, Gen. Rel. Grav. **42**, 2633 (2010).
- [6] S. A. Pavluchenko and A. V. Toporensky, Mod. Phys. Lett. **A 24**, 513-521 (2009).
- [7] S. A. Pavluchenko, Phys. Rev. **D 80**, 107501 (2009).
- [8] D. Chirkov, S. Pavluchenko and A. Toporensky, Mod. Phys. Lett. **A 29**, 1450093 (2014).
- [9] V. D. Ivashchuk, Grav. Cosmol. **16**, 118 (2010).
- [10] V. D. Ivashchuk, Int. J. Geom. Meth. Mod. Phys. **7**, 797 (2010).
- [11] V. D. Ivashchuk and A. A. Kobtsev, Eur. Phys. J. **C 75**, 177 (2015).
- [12] S. A. Pavluchenko, Phys. Rev. **D 92**, 104017 (2015).
- [13] K. K. Ernazarov, V. D. Ivashchuk and A. A. Kobtsev, Grav. Cosmol. **22**, 245 (2016).
- [14] V. D. Ivashchuk, On stability of exponential cosmological solutions with non-static volume factor in the Einstein-Gauss-Bonnet model; arXiv: 1607.01244v2.
- [15] A. G. Riess et al. Astron. J. **116**, 1009 (1998).
- [16] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999).
- [17] M. Kowalski, D. Rubin et al., Improved cosmological constraints from new, old and combined supernova datasets; arXiv: 0804.4142.
- [18] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. **571**, A1 (2014).
- [19] M. Rainer and A. Zhuk, Gen. Relativ. Gravit. **32**, 79-104 (2000); gr-qc/9808073.
- [20] V.D. Ivashchuk and V.N. Melnikov, Grav. Cosmol. **2**(3), 211-220 (1996); hep-th/9612054.
- [21] K. A. Bronnikov, V. D. Ivashchuk and V. N. Melnikov, Nuovo Cimento **B 102**, 209 (1998).
- [22] V. D. Ivashchuk and V. N. Melnikov, Grav. Cosmol. **20**, 26 (2014).
- [23] E. V. Pitjeva, Astron. Vestnik **47**, 419 (2013).